Modeling of Thermal-Mechanical Instabilities: from Macroscale to Microscale

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Part I: Thermoelastic Instability in Automotive Brakes and Clutches
An automotive disk brake system consists of a rotating disk sliding between two friction pads.
Hot spots development on the sliding surface. The resulting thermal stresses can cause brake judder and cracking/plastic deformation.

Brake dynamometer with an infrared sensor system

$Ob, t=1\ s, T_m=93^\circ C, \Delta T=31^\circ C$
Disc surface temperature field
Automotive clutches

Friction material

R - rotating part
S - stationary part

Pressure plate
End plate
Hot spots on clutches

Hot spotting after single engagements of a clutch

Hot spotting after multiple engagements of a clutch
Hot spots on heavy duty clutches

(a) large focal hot spots around circumference   (b) hot spots in both radius/circumference

“scars” left by hot spots on clutch discs
Hot spots in high speed sliding systems are caused by an unstable interaction between thermoelastic distortion and frictional heat generation. It is therefore also known as frictionally-excited ThermoElastic Instability or TEI (Barber, 1969).

Conventional design practice for brakes and clutches argues that if the contact pressure is non-uniform, wear will occur preferentially in the high pressure regions and restore uniformity.

However, non-uniform pressure implies a correspondingly non-uniform rate of frictional heat generation and high pressure regions will expand outwards increasing the non-uniformity (positive feedback).
Mechanism of TEI

Contact pressure $p(x,y,t)$ and contact area

Solution of contact problem

Unconstrained thermal distortion

Frictional heat generation

Frictional heat generation $q(x,y,t) = fVp(x,y,t)$

Solution of heat conduction problem

$V =$ sliding speed
$f =$ friction coefficient

This feedback implies the possibility of instability if the gain is sufficiently high.
Applications of TEI theory

Frictionally Excited TEI

- railway
- aircraft
- seals & bearings
- automobile
Earlier work (Lee 1993; Yeo 1996) considered a layer sliding between two half planes. This can be seen as a rough approximation to a disc sliding between two pads.

Two distinct modes are obtained. One is symmetric with respect to the mid-plane and one is antisymmetric.
The same perturbation method was extended to general geometries by Yi et al. (2000, 2002, 2006) using the finite element method.

If the system is unstable, there must exist one or more modes (perturbations) that can grow exponentially with time, permitting a description of the temperature field in the form

$$T(x, y, z, t) = \Theta(x, y, z)e^{bt}.$$  

If we substitute this and similar expressions for the elastic displacements into the governing equations (equations of heat conduction and thermoelasticity) and the boundary conditions, the common exponential factor can be cancelled.
The governing heat conduction equation, stress-strain constitutive law, the conforming contact condition and the frictional heat generation equation can be written in the perturbed forms.

Discretization of the equations yields an eigenvalue problem

\[ H^{-1}(fVA - K - C)\Theta = b\Theta \]

- \( n \) – Fourier number (number of hot spots)
- \( V \) – sliding speed
- \( b \) – exponential growth rate (eigenvalue)
- \( \Theta \) – nodal temperature
Instability is indicated if there exists at least one eigenvalue with positive real part. The corresponding unstable eigenmode defines a clearly identifiable pattern of hot spots on the sliding surfaces.

This is the basis of the HotSpotter® software package, which is now in use for automotive brake and clutch design/research in a number of companies and institutions.
HotSpotter® user interface

- **Menu bar**
- **Message Box**
- **Figure body**
- **Buttons**
Parameter input dialogue window

1st Column: Boundary condition, etc.

2nd Column: Geometric parameters

3rd Column: Material properties
Computational models of simplified clutch systems developed in *Hotspotter*: single disc clutch (left) and multidisc clutch (right).
Cross sectional plane of a brake model (it contains the hat section, vent section and pad section)

3-D finite element mesh of a brake model in HotSpotter®
HotSpotter accepts a script-based input file, which is similar to the data format used in ABAQUS.

It provides users with full freedom to specify boundary conditions, geometry shape, meshing, element type and surface definition.

```plaintext
*node
104, 44.5e-3, 0.67e-3
...
*element, type=2d
100,100,101,201,200
...
*elset
*material, name=fric
*elastic
3.1e8, 0.3
*conductivity
...
*friction
*boundary
*step
...
*end step
```
Dominant mode patterns

Each mode has a critical sliding speed above which the mode becomes unstable.
The temperature profile across the thickness of the disc stack of a clutch shows the dominant anti-symmetric mode shape with a higher temperature variation near the middle.
From the design point of view, a higher critical speed of TEI is preferred. The critical speed will increase if we

- increase the friction layer thickness
- increase the friction material thermal conductivity
- reduce the friction layer rigidity
- reduce the rotor radial width
- reduce the rotor thickness
- increase the rotor thermal conductivity
Challenges

• Incorporation of convective term and nonconforming contact

• Better eigenvalue solvers are needed for unsymmetrical complex matrices

• Integration of squeal and TEI formulations

• Development of an interface to major commercial software
Part II: Thermoelastic Damping (TED) in MEMS
Resonant sensors and filters are important MEMS (microelectromechanical systems) and NEMS (nanoelectromechanical systems) devices that can resonate at certain ranges of radio frequencies. They are widely used in the
(1) internet technology
(2) mechanical and biomedical sensors
(3) digital electronics
for making accurate frequency comparisons and for generating narrowband frequencies in the microwave region.

However, operation of MEMS/NEMS resonators and filters at high frequencies usually involves damping-related energy dissipation processes. Minimization of the energy loss or achieving high quality factor (Q-factor) is often a key design objective.
There are various damping sources in MEMS/NEMS. The investigation of thermoelastic damping (TED) in MEMS/NEMS is relatively very recent, being prompted by the pursuit of low energy dissipation in designing and fabricating high-precision sensors and filters (Lifshitz and Roukes 2000).

TED is a phenomenon related to the irreversible heat dissipation induced by the coupling between heat transfer and strain rate during the compression and decompression of an oscillating system.
Transient simulations of energy dissipation in MEMS/NEMS (computationally determine the energy loss per cycle of resonance) is possible but numerically difficult.
Governing differential equations

Heat transfer equation

\[ k \nabla^2 T = \frac{\partial T}{\partial t} + \frac{E \alpha T_0}{(1-2\nu)C_v} \frac{\partial (\varepsilon_x + \varepsilon_y + \varepsilon_z)}{\partial t} \]

Equation of motion in the differential form

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= \rho \frac{\partial^2 u_x}{\partial t^2} \\
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= \rho \frac{\partial^2 u_y}{\partial t^2} \\
\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= \rho \frac{\partial^2 u_z}{\partial t^2}
\end{align*}
\]

Stress-strain relationship: \( \sigma = C(\varepsilon - \varepsilon_t) \)
Perturbation method

Perturbation assumption

\[
\begin{align*}
T &= \Re\{e^{bt} \Theta\} \\
u &= \Re\{e^{bt} U\} \\
u' &= \Re\{e^{bt} U'\}
\end{align*}
\]

Note: a second-order eigenvalue equation would be present if the velocity were not expressed in a perturbed form.

Matrix equation for heat transfer

\[(K + bH)\Theta + bFU = 0\]

Equation of motion in the matrix form

\[LU - G\Theta + bMU' = 0\]

Displacement-velocity relationship

\[U' = bU\]
Eigenvalue formulation

Eigenvalue equation:

\[ \tilde{\Lambda}X = b\tilde{\mathbf{B}}X, \]

\[ \tilde{\Lambda} = \begin{bmatrix} K & 0 & 0 \\ G & -L & 0 \\ 0 & 0 & I \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} H & F & 0 \\ 0 & 0 & M \\ 0 & I & 0 \end{bmatrix} \]

\[ X = [\Theta, U, U']^T \]

The eigenvalue of the equation is the growth rate \( b \) and the eigenvector is a vector containing the nodal temperature, displacement and velocity.
Eigenmodes for flexural-mode vibration

Each resonant mode is associated with a resonant frequency and a Q-value.
Beam model (dimensionless Q-factor vs dimensional beam thickness)

Effect of the radial width in the 3-D ring model
Using higher order elements

Comparisons between linear elements and quadratic elements for Computing thermoelastic damping
Q-factor vs radius of a circular disc

Left graph: Q-factor vs radius (nm) for different values of \( m \).

Right graph: Q-factor vs \( \frac{R_i}{R_o} \) for different values of \( R_o \) and \( m \).
Results for an elliptical plate resonator
Eigenmodes (contour-mode vibration)

Temperature profiles for dominant eigenmodes in in-plane vibration of elliptical thin plates
Micro mirrors

Micro-mirror design with crab-shaped springs connecting the central mirror to the peripheral posts.
Models for micro mirrors

Evaluation of thermoelastic damping in four different designs of micro mirrors. It has been found that model B with an included angle of 90° between the peripheral beam and the central mirror exhibits the highest Q value and is therefore the best design among the four models.
Resonant frequency and Q-factor as a function of the mirror size
MEMS sensor experiment

MEMS experiment performed at DU (Rahafrooz, Pourkamali 2009)
Experimental and simulation results for the Q-factor as a function of the beam width and length under the collective effect of thermoelastic damping and fluid damping.
Table 2: Comparisons of resonant frequency between experiments and theoretical predictions

<table>
<thead>
<tr>
<th>Beam length (µm)</th>
<th>Finite element freq (MHz)</th>
<th>Analytical solution Freq (MHz)</th>
<th>Test freq (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>6.915</td>
<td>6.945</td>
<td>5.438 ± 0.149</td>
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<tr>
<td>150</td>
<td>3.17</td>
<td>3.087</td>
<td>2.679 ± 0.055</td>
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<tr>
<td>200</td>
<td>1.810</td>
<td>1.737</td>
<td>1.615 ± 0.070</td>
</tr>
</tbody>
</table>
Table 3: Comparisons of quality factor between experiments and theoretical predictions

<table>
<thead>
<tr>
<th>Beam length (μm)</th>
<th>Beam width (μm)</th>
<th>Experiment Q factor</th>
<th>Predicted Q based on the experimentally determined γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
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<td>295</td>
<td>423</td>
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<tr>
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<td>10</td>
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We are currently investigating the effect of geometry on thermoelastic damping.
When the device size approaches nanoscale, the material properties especially the thermal conductivity will change due to the constraints imposed on the propagation of phonons.

For silicon, the mean free path of phonons at room temperature is about 43 nm. Above this threshold, the thermal conductivities in both thickness and lateral directions can be estimated by an approximation formula provided by Flik et al. (1992)
Fick’s laws of diffusion do not consider the finite speed of thermal waves. On the nanoscale the Fourier law of heat conduction will need to be extended to the most general situation involving heat flux and its first time derivative, i.e.

$$ q + t_0 \frac{\partial q}{\partial t} = -k \nabla T $$

where $q$ is the heat flux, $t_0$ is the relaxation time and $k$ is the thermal conductivity.
Nanoscale effects: heat conduction

The coupled heat conduction equation becomes

\[ \nabla \cdot (\nabla T) = \frac{\rho c}{k} \left( \frac{\partial T}{\partial t} + t_0 \frac{\partial^2 T}{\partial t^2} \right) + \frac{E \alpha}{k(1-2\nu)} T_0 \sum_{i=1}^{3} \left( \frac{\partial u_i}{\partial x_i} + \frac{\partial^2 u_i}{\partial x_i^2} \right) \]

where \( \rho \) is the density, \( c \) is the specific heat, \( \alpha \) is the thermal expansion coefficient, \( E \) is Young’s modulus, \( T_0 \) is the mean temperature and \( u \) is the displacement. Unlike in the classical theory, this equation is of hyperbolic type and predicts a finite speed for heat propagation, provided \( t_0 > 0 \). In addition, it was shown that \( t_0 \) can be quantitatively estimated in terms of measurable macroscopic parameters as

\[ t_0 = \frac{3k}{\nu^2 \rho c} \]

where \( \nu \) is the phonon velocity.
A comparison between the classical TED solution and the solution considering the finite thermal wave speed on nanoscale.

Nanoscale effects: Q factor
In the future work we would like to answer:
(i) What is the effect of dimensionality and geometry on the finite wave propagation and the associated TED?
(ii) What is the role of finite wave speed of thermal disturbance in the longitudinal and torsional oscillation modes?
(iii) How will the TED interact with fluid damping and electrostatics?
References

References

Acknowledgments

- Dr. S. Pourkamali, Associate Professor, ECE, University of Texas
- Dr. M. Matin, Associated Professor, ECE, DU
- Dr. J. R. Barber, Professor, ME, University of Michigan
- Dr. P. Zagrodzki, Senior Research Engineer, Raybestos Co.
- Dr. Y. H. Jang, Professor, Yonsei University, Korea
- D. Hartsock, Senior Engineer, Ford Motor Company (retired)
- X. Guo, Ph.D. student, MME, DU
- Z. Chen, M.S. student, MME, DU
- A. Rahafrooz, Ph.D. student, ECE, DU
- H. Tang, Ph.D. student, ECE, DU